Dashboard Mechanisms for Online Marketplaces[1]

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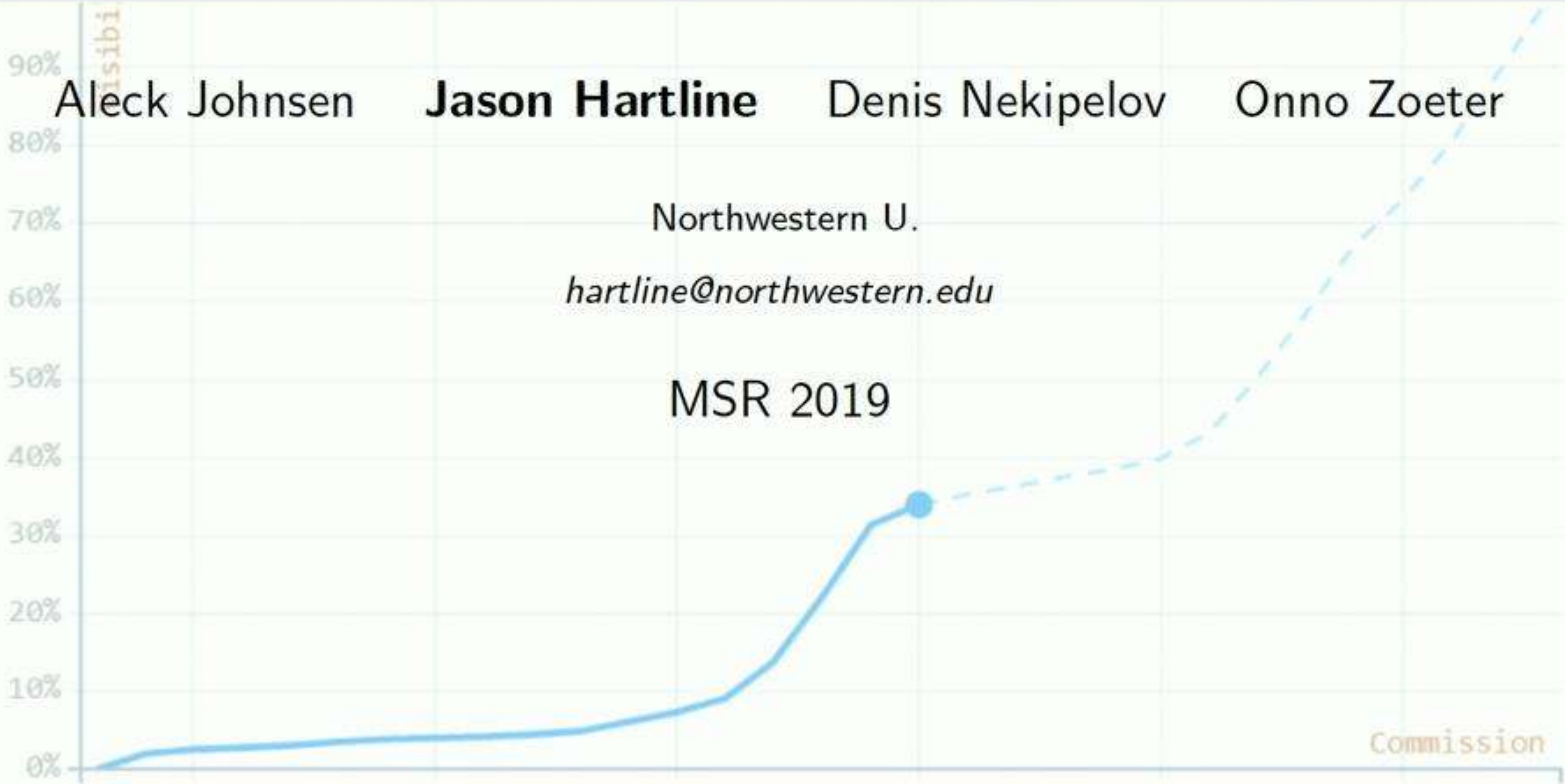

[1] https://arxiv.org/abs/1905.05750

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Examples: ad auctions, booking.com, eBay, etc.

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- VCG mechanism, cf. second-price auction. [Vickrey '61, Clarke '71, Groves '73]
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- agents can easily optimize bids for dashboard. what dashboard?
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Informal Definition (Dashboard Mechanism)

The dashboard mechanism is (for given dashboard and allocation alg):

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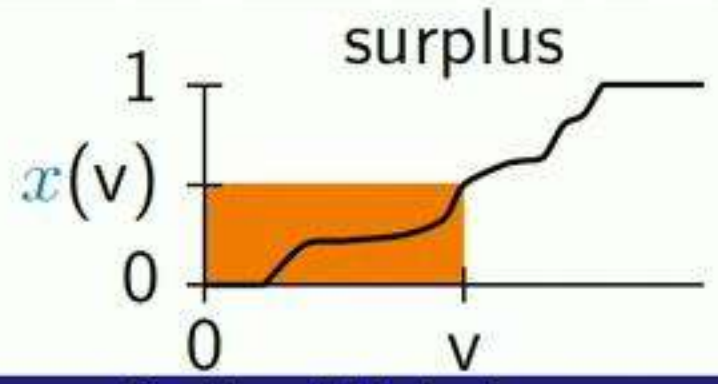

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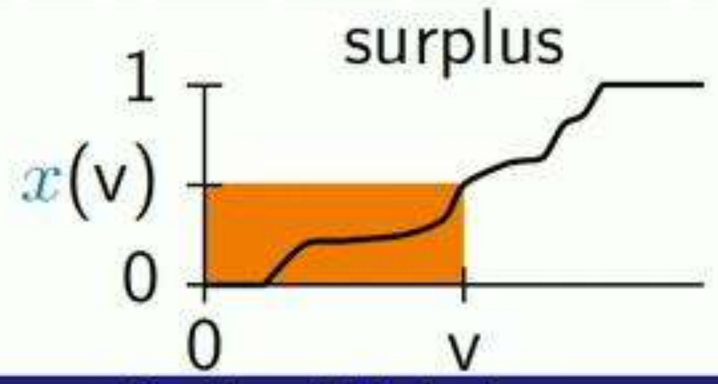

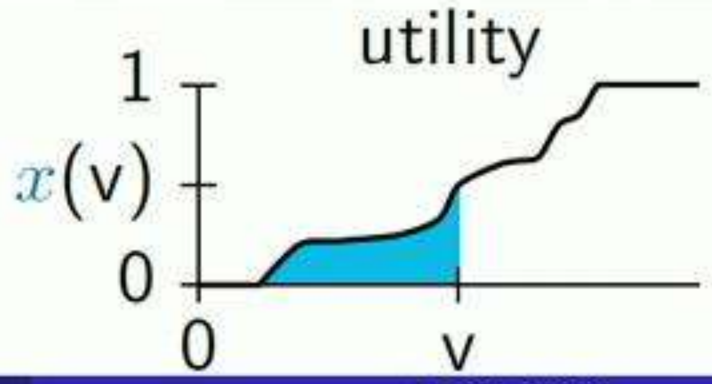

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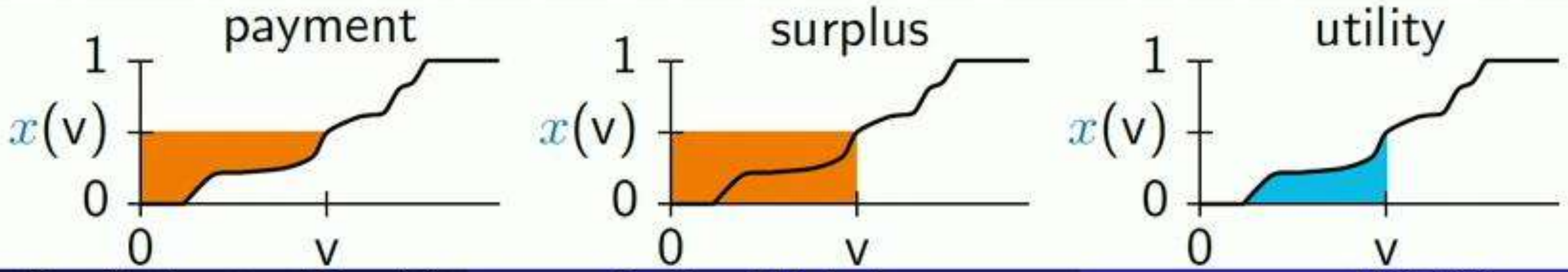

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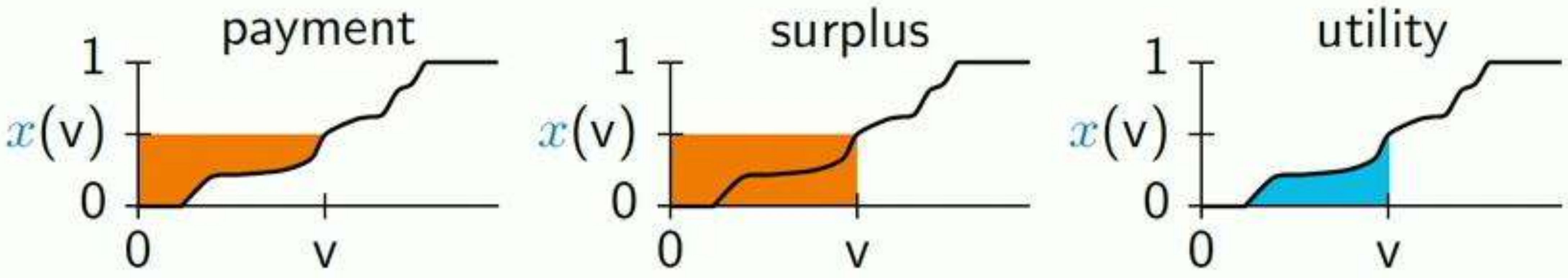

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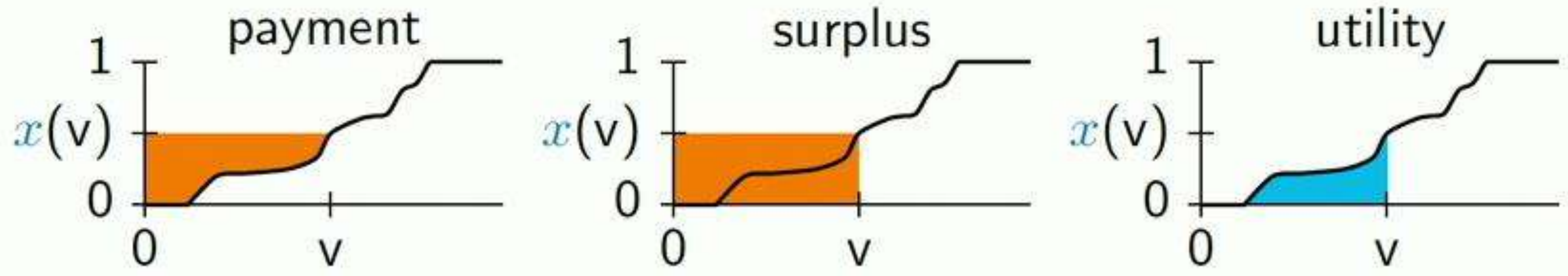

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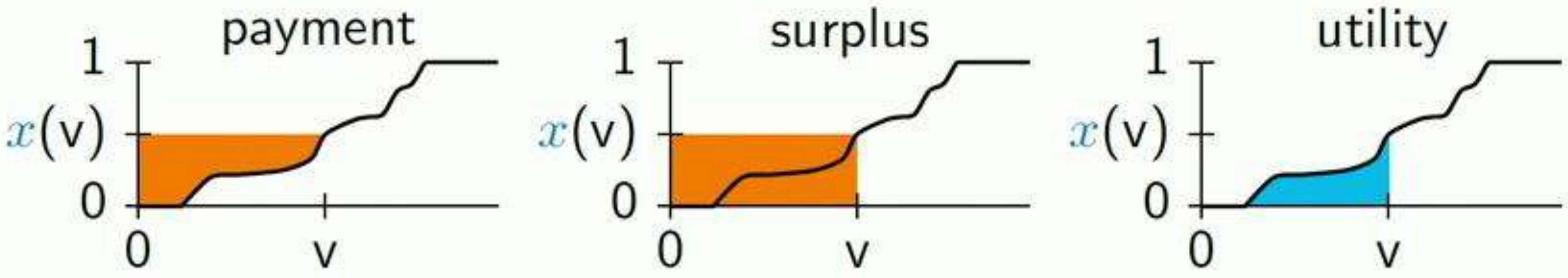

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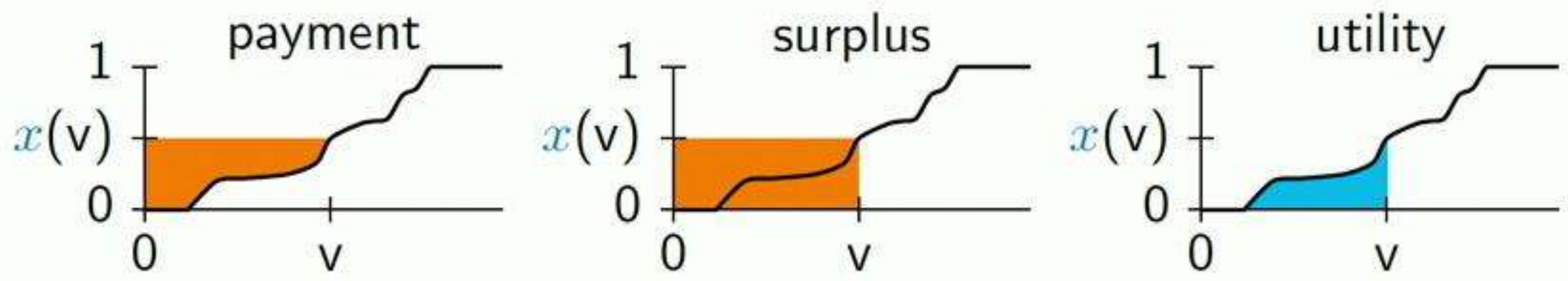

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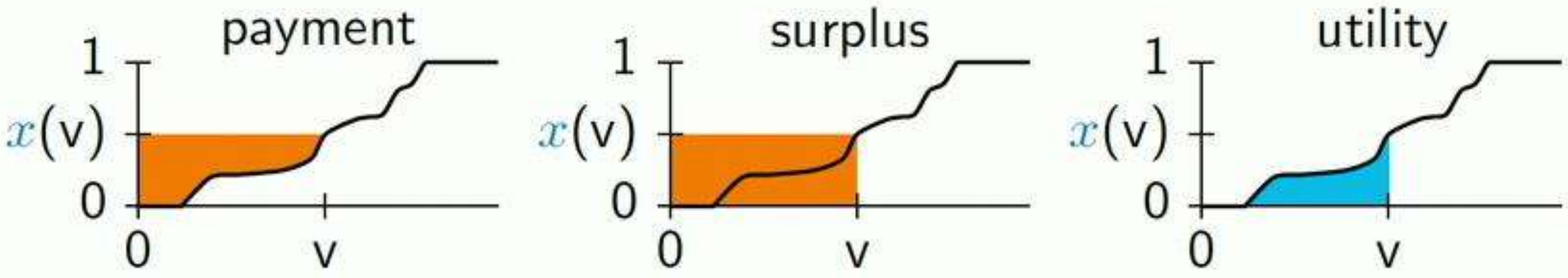

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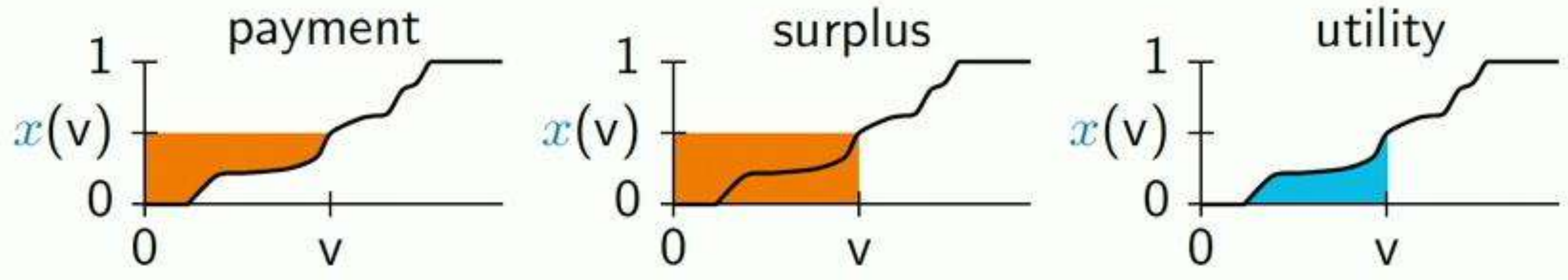

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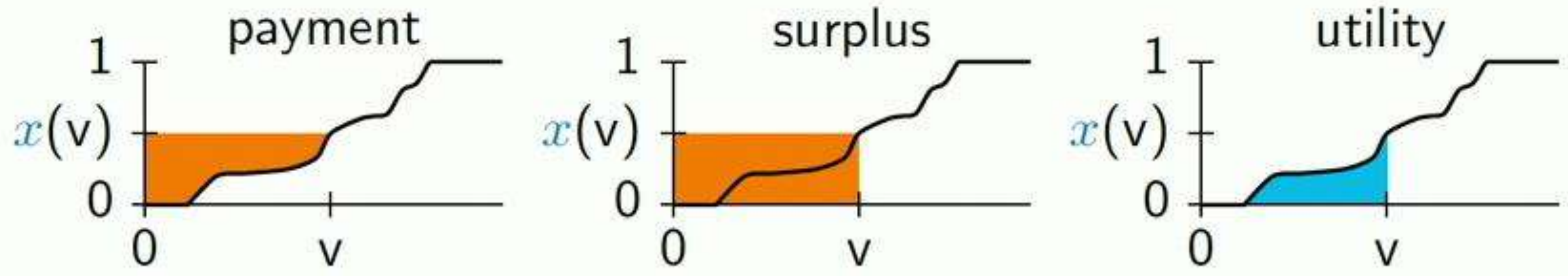

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The single-agent winner-pays-bid implementation of x is $\tilde{x}$ satisfying:

- agent's bid strategy:* $b(\mathrm{v}) = p(\mathrm{v})/x(\mathrm{v}) = \mathrm{v} - \frac{1}{x(\mathrm{v})} \int_0^{\mathrm{v}} x(z)\, dz + \frac{p(0)}{x(\mathrm{v})}$.
- bid allocation rule: $\tilde{x}(\mathrm{b}) = x(b^{-1}(\mathrm{b}))$

* monotone iff $p(0) \leq 0$

Main Challenge: inverting bids in multi-agent settings.

Outline

1. Introduction and Motivation
2. Single-agent Winner-pays-bid Mechanisms
3. Dashboard Construction and Analysis
4. Single-call Dashboard Mechanisms
5. Discussion and Directions

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Model

Dynamic model:

- dynamic iterated environment, in stage $s \in \{1, \dots, t\}$:
  - n agent valuation profile: $\mathbf{v}^{(s)} = (v_1^{(s)}, \dots, v_n^{(s)})$
  - stochastic allocation algorithm: $\boldsymbol{x}^{(s)} : \mathbb{R}^n \to [0,1]^n$.
- linear utility: $u_i = \sum_{s=1}^{t} [v_i^{(s)} x_i^{(s)} - p_i^{(s)}]$.
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Note: for single and and single stage analysis, will drop i and s.

Dashboard Mechanisms

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Definition (Dashboard Mechanism)

The dashboard mechanism $\tilde{\boldsymbol{x}} : \mathbb{R}^n \to [0,1]^n$ for dashboard $\tilde{\boldsymbol{y}}$, algo $\boldsymbol{x}$ is:

1. publish dashboard $\tilde{\boldsymbol{y}}$; solicit bids $\mathbf{b}$.
2. infer values $\hat{\mathbf{v}}$ as $\hat{v}_i = c_i^{-1}(b_i)$ for $\tilde{y}_i$.
3. run $\boldsymbol{x}$ on $\hat{\mathbf{v}}$ to get allocation $\mathbf{x} \sim \boldsymbol{x}(\hat{\mathbf{v}})$ and payments $\mathbf{p}$ as $p_i = b_i x_i$.

Last-stage Dashboard

Definition (Last-stage Dashboard)

In stage s:

- last stage inferred values: $\hat{\mathbf{v}}^{(s-1)}$
- last stage allocation rules: $\boldsymbol{y}^{(s)}$ as $y_i^{(s)}(z) = x_i^{(s-1)}(z, \hat{\mathbf{v}}_{-i}^{(s-1)})$.
- dashboard: $\tilde{\boldsymbol{y}}^{(s)}$ with $\tilde{y}_i^{(s)}$ corresponding to $y_i^{(s)}$.

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Theorem (Static Analysis)

For last-stage dashboard and static environment, the action profile for "follow the dashboard" strategy converges (in two rounds) to Nash equilibrium of stage game.

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Goal: dashboard robust to changing values and environment.

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Intuition for Payment Rebalancing

Goal: dashboard robust to changing values and environment.

Intuition: For empirical values dashboard:

- When $x_i^{(s)} \neq x_i^{(s-1)}$:
  - allocation is correct (for estimated value $\hat{v}_i^{(s)}$)
  - payment is incorrect.
- calculate error in payment (positive or negative), add to balance.
- future dashboards include lump sum payment to reduce balance.

Payment Rebalancing Dashboards

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- For $\eta \leq y(0)$, the balance resolved in winning stage is in $[\mathsf{L}\,\eta, \mathsf{L}]$.
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Corollary (Dynamic Analysis)

For payment rebalancing dashboard and dynamic environment, seq. dashboard mech. is $\frac{\bar{\mathsf{v}}}{\eta\,t}$-approx. strat. equiv. to seq. truthful mech.

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Single-call Algorithms

Definition (Single-call Model)

Only access to algorithm $\boldsymbol{x}$ is by implementing its outcome, i.e., $\mathbf{x} \sim \boldsymbol{x}(\hat{\mathbf{v}})$.

Recall Motivation: online markets:

- short-lived users matched to long-lived agents.
- matching algorithm, e.g.:
  - marketplace prioritize agents
  - users select agents

Discussion & Directions

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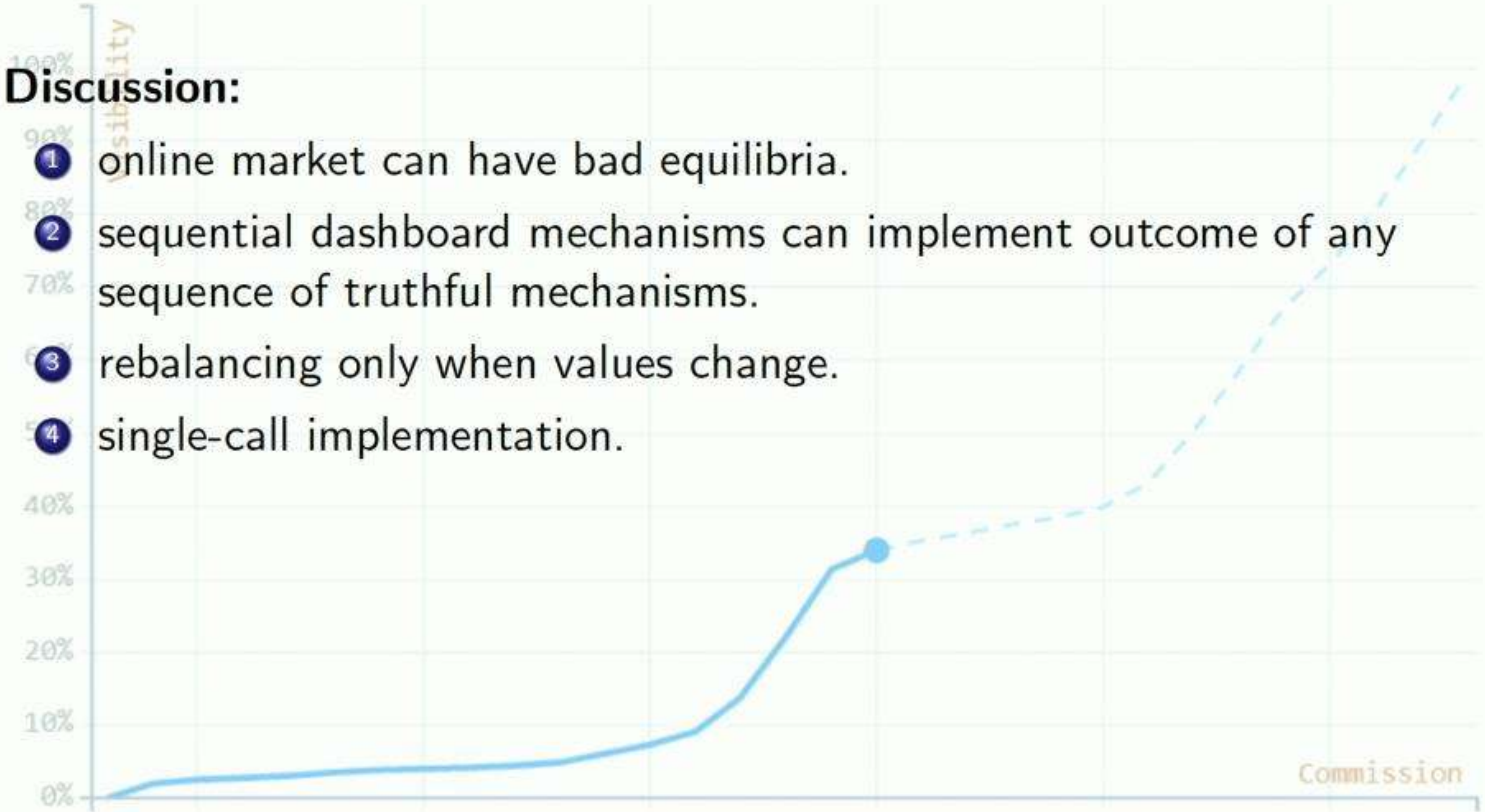

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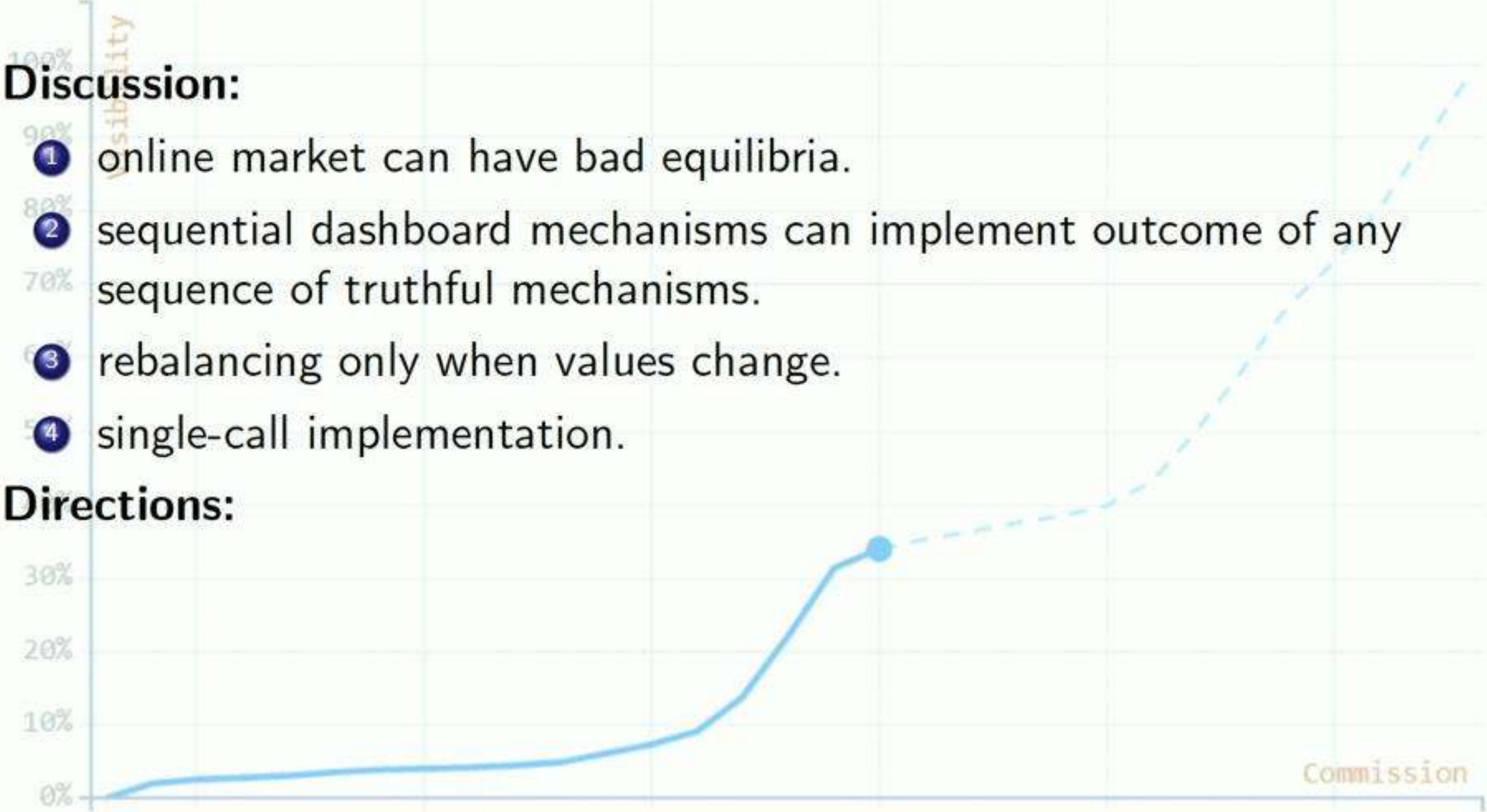

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- broad theory for non-truthful mechanism design. [cf. Hartline, Taggart '19]